II. Mr. C. F. GUMMER solved the problem in a similar manner and added: It may be of interest to find the expectation. This will be

$$\sum_{r=k}^{r=N-n+k} \frac{\binom{r-1}{k-1}\binom{N-r}{n-k}}{\binom{N}{n}} r = \frac{k\,\lfloor n}{\lfloor N} f(N, n, k),$$

where r is the number on the kth ball, and

$$f(N, n, k) = \sum_{r=k}^{r=N-n+k} \binom{N-n}{r-k} \frac{\lfloor r}{\lfloor k} \frac{\lfloor N-r}{\lfloor n-k}.$$

From the relation

$$\binom{N-n}{r-k} = \binom{N-n-1}{r-k-1} + \binom{N-n-1}{r-k},$$

it follows that

$$f(N, n, k) = (n-k+1)f(N, n+1, k) + (k+1)f(N, n+1, k+1). \tag{1}$$

Now $f(N, N, k) = 1$, being independent of k. Therefore, by (1), $f(N, N-1, k) = N+1$, also independent of k, and finally $f(N, n, k) = (N+1)N\cdots(n+2)$. Hence, the expectation for the kth ball is $k(N+1)/(n+1)$.

2699 [May, 1918]. Proposed by the late ROGER E. MOORE, University of Wisconsin.

Show that if $a_k^{(r)}$ denotes the kth term of an arithmetic progression of order r, and c_k denotes the kth binomial coefficient in the expansion of $(a-b)^n$, n being a positive integer,

$$s \equiv \sum_{k=1}^{n+1} c_k a_k^{(r)} = 0, \qquad \text{if} \qquad n > r.$$

SOLUTION BY ELBERT H. CLARKE, Hiram College.

Let d_0 be the first term in the arithmetic progression and let $d_1, \cdots, d_r$ denote the initial difference of each order. Using the usual abbreviated notation for binomial coefficients, we write

$$a_k^{(r)} = \binom{k-1}{0} d_0 + \binom{k-1}{1} d_1 + \binom{k-1}{2} d_2 + \cdots + \binom{k-1}{r} d_r,$$

$$c_k = (-1)^{k-1}\binom{n}{k-1}$$

and

$$\sum_{k=1}^{n+1} c_k a_k^{(r)} = \sum_{i=0}^{r} d_i \sum_{k=1}^{n+1} (-1)^{k-1} \binom{k-1}{i}\binom{n}{k-1}.$$

Consider the inner sum. Since $\binom{k-1}{i} = 0$, for $k < i+1$,

$$\sum_{k=1}^{n+1} (-1)^{k-1} \binom{k-1}{i}\binom{n}{k-1} = \sum_{k=i+1}^{n+1} (-1)^{k-1} \binom{k-1}{i}\binom{n}{k-1},$$

and the latter expression easily becomes

$$\binom{n}{i} \sum_{k=i+1}^{n+1} (-1)^{k-1} \binom{n-i}{k-(i+1)}.$$

Now put $k - i - 1 = t$ and we have

$$\binom{n}{i} \sum_{t=0}^{n-i} (-1)^{t+i} \binom{n-i}{t}.$$

But the expression under summation is simply $(-1)^i(1-1)^{n-i}$. Hence, the coefficient of every d_i is zero. Therefore,

$$\sum_{k=1}^{n+1} c_k a_k^{(r)} = 0, \qquad n > r.$$

NOTE.—This proof is good only for $n > r$. Numerical examples can easily be given to show that the formula is not true for $n \leqq r$.

Also solved by HORACE L. OLSON.

NOTES AND NEWS.

EDITED BY E. J. MOULTON, Northwestern University, Evanston, Ill.

Dr. J. N. VAN DER VRIES has resigned his position as professor of mathematics at the University of Kansas to continue his work as secretary of the central district of the Chamber of Commerce of the United States, with headquarters at Chicago.

Mr. G. H. CRESSE, previously instructor in the University of Michigan, has been appointed to an instructorship in mathematics at the U. S. Naval Academy at Annapolis; he was granted the degree of doctor of philosophy at the University of Chicago in December.

Mr. F. S. NOWLAN, of Bowdoin College, has been promoted to an assistant professorship of mathematics.

Dr. MARION B. WHITE, formerly of the Ypsilanti State Normal College, Michigan, is now professor of mathematics at Carleton College.

Dr. TOBIAS DANTZIG and Dr. G. A. PFEIFFER have been appointed instructors in mathematics at Columbia University.

Professor GRIFFITH C. EVANS is now scientific attaché to the American Embassy at Rome.

Dr. W. GROSS, of the University of Vienna, has been promoted to professor of mathematics.

Professor E. WIECHERT of Göttingen has been appointed professor of geodesy and geophysics at the University of Berlin.

Efforts are being made to establish a chair of mathematical physics at the University of Edinburgh in memory of the late Professor TAIT.

At the University of Strasbourg Professor RENÉ M. FRÉCHET, of the University of Poitiers, has been appointed professor of mathematics, and PIERRE WEISS, professor at the Polytechnikum, Zurich, professor of general physics.

CHARLES L. DOOLITTLE, professor emeritus of astronomy at the University of Pennsylvania, and director of the Flower Observatory, died on March 3, 1919, aged seventy-five years.